BIOL/PHYS 438

Zoological Physics

- **Logistics**
- *Review* of the **physics** of **Waves**
  - Basic phenomenology: period & wavelength
  - SHM in time and space: "the" Wave Equation
  - Phase *vs.* Group velocities: Dispersion
  - Reflection & Refraction: Snell & TIR
  - Electromagnetic waves: spectrum, color
  - Thin Film Interference: butterfly colors

Logistics

Assignment 1: Solutions now online!

Assignment 2: Solutions online soon!

Assignment 3: Solutions online soon!

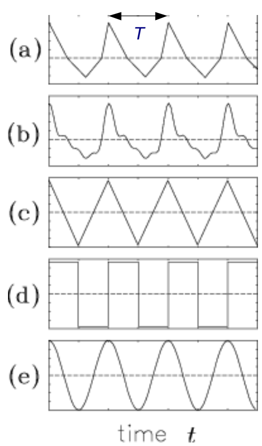
Assignment 4: due **next Tuesday**

Assignment 5: due **Thursday after next**

Jess will be away next week (5-9 March)

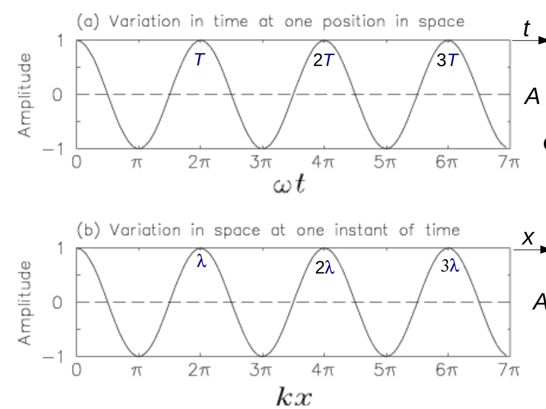
Hopefully your **Projects** are underway by now . . .

Periodic Phenomena



Period T

Sinusoidal Wave



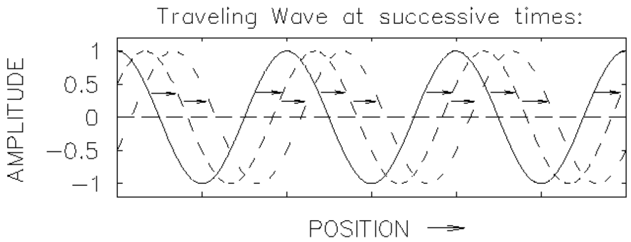
$A \cos \omega t + \phi$

$\omega = 2\pi/T$

$A \cos kx + \phi$

$k = 2\pi/\lambda$

Phase Velocity



$y(t) = A \cos(kx - \omega t + \phi)$

$c = \lambda / T = \omega / k$

"The" Wave Equation

Suppose we know that we have a *traveling wave* $A(x,t) = A_0 \cos(kx - \omega t)$.

At a *fixed position* $(x = \text{const})$ we see *SHM* in time:

$$\left(\frac{\partial^2 A}{\partial t^2}\right)_x = -\omega^2 A \tag{8}$$

(Read: "The second partial derivative of A with respect to time [*i.e.* the *acceleration* of A] with x held fixed is equal to $-\omega^2$ times A itself.") *I.e.* we must have a *linear restoring force.*

"The" Wave Equation, cont'd

Similarly, if we take a "snapshot" (hold t fixed) and look at the *spatial* variation of A, we find the oscillatory behaviour analogous to *SHM*,

$$\left(\frac{\partial^2 A}{\partial x^2}\right)_t = -k^2 A \tag{9}$$

(Read: "The second partial derivative of A with respect to position [*i.e.* the *curvature* of A] with t held fixed is equal to $-k^2$ times A itself.")

"The" Wave Equation, cont'd

Thus $A = -\frac{1}{\omega^2}\left(\frac{\partial^2 A}{\partial t^2}\right)_x = -\frac{1}{k^2}\left(\frac{\partial^2 A}{\partial x^2}\right)_t.$

If we multiply both sides by $-k^2$, we get

$$\frac{k^2}{\omega^2}\left(\frac{\partial^2 A}{\partial t^2}\right)_x = \left(\frac{\partial^2 A}{\partial x^2}\right)_t.$$

But $\omega = ck$ so $\frac{k^2}{\omega^2} = \frac{1}{c^2}$, giving

the **Wave Equation**

$$\boxed{\frac{\partial^2 A}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0} \tag{10}$$

In words, the *curvature* of A is equal to $1/c^2$ times the *acceleration* of A at any (x,t) point.

Other "Wave Equations"

"The" Wave Equation governs our two most important types of waves:

SOUND (vibrations of a compressible medium) and **LIGHT** (electromagnetic oscillations), for which the *phase* and *group* velocities are the same:

$$\omega \approx c\,k$$

But there are others for which this is not true:

WATER WAVES: $\omega = \sqrt{\frac{g\,k}{2}}$ and

MATTER WAVES, (see Schroedinger Equation)

Group Velocity

The ***phase velocity*** of a wave is always given by

$$v_{ph} = \omega / k$$

But ***information*** travels at the **group velocity**:

$$v_{\rm g} \equiv \frac{\partial \omega}{\partial k}$$

These are not always the same!

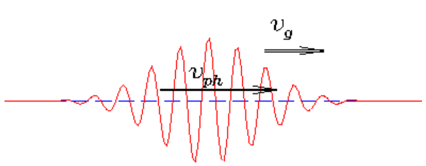
Water Waves

For **DEEP OCEAN WATER WAVES**,

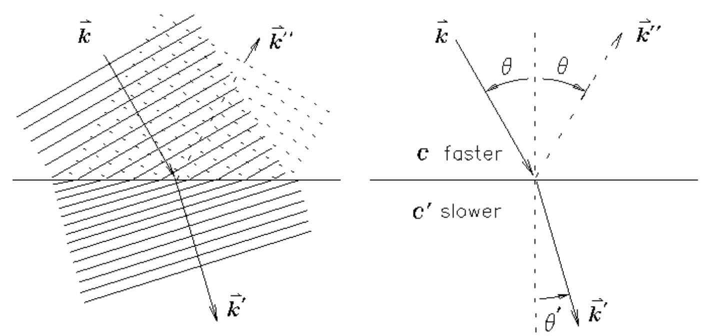
$$\omega = \sqrt{\frac{g\,k}{2}}$$

giving

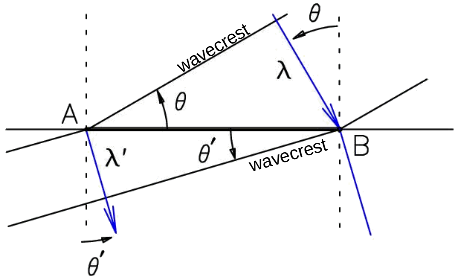
$$v_{\rm ph} = \sqrt{\frac{g}{2k}} \quad \text{and} \quad v_{\rm g} = \frac{1}{2}\sqrt{\frac{g}{2k}}.$$



Reflection & Refraction

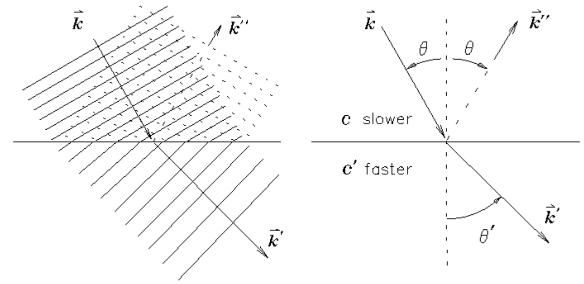


Snell's Law



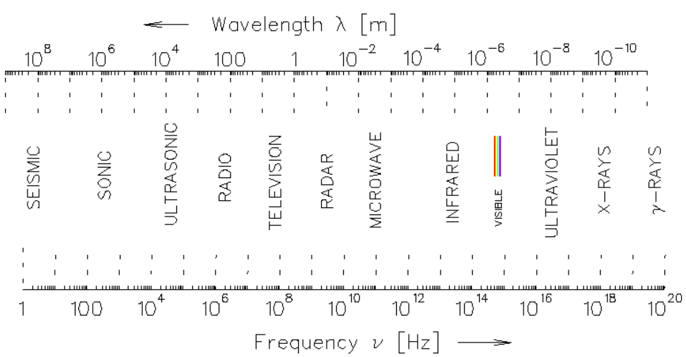
Dissimilar triangles: $\sin\theta / \sin\theta' = \lambda/\lambda' = c/c' = n'/n$

Total Internal Reflection



$\sin\theta/\sin\theta' = n'/n$: For $n' > n$, at some angle θ_c this predicts $\theta' = \pi/2$, *i.e.* there is **no** refracted wave; then for $\theta > \theta_c$ we get a *perfect mirror*! (Ask any fish!)

The Electromagnetic Spectrum



Colors

How do we *perceive* color?

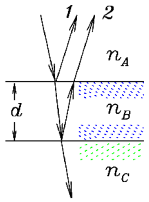
Can you tell a pure green laser from a mixture of pure blue and pure yellow lasers?

Can you tell a pure violet laser from a mixture of pure red and blue lasers?

What is the difference between violet and "purple"?

Thin Film Interference

We always draw the reflected and refracted rays at a small angle to the normal so that the two reflected rays (***1*** & ***2***) can be shown separately; but in reality we are always talking about **normal incidence**.



To decide if rays ***1*** & ***2*** are **in phase** or **out of phase**, we **add up their phase differences**. Upon **reflection**, if $n_B > n_A$, ray ***1*** experiences a phase shift of π ; ray ***2*** has a similar phase shift if $n_C > n_B$. Then the path length difference (2d) gives a phase difference of $\Delta\theta_{path} = 2\pi(\Delta\ell/\lambda_B)$ where λ_B is the wavelength in medium B. Let's suppose $n_C > n_B > n_A$ so that both reflected rays get the same "phase flip". Then the path length difference of $2d$ is the only source of $\Delta\theta = 2\pi(2d/\lambda_B)$.

If $d = \lambda_B/4$ (what we call a "**quarter-wave plate**") then rays ***1*** & ***2*** will interfere **destructively**, giving **minimum reflection** & **maximum transmission**. This is used in "anti-glare" coatings on windows, glasses and camera lenses.