BIOL/PHYS 438

Zoological Physics

- Logistics
- Corrections from last week
- Review of Mechanics
- Introduction to Entropy and Temperature
- Back to Ch. 2: Energy Management

Logistics

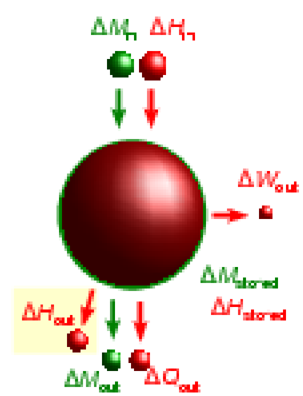
Assignment 1:

- Login and Update your Profile!
- Please Email us about yourself!

Notation: a few symbols

X – Any abstract quantity
dX – An infinitesimal change in x
ΔX – A finite change in x
t – Time [s], x, y, z, d, r, R – Distances [m] (usually)
M – Mass [kg]
U – Energy [J] (usually potential energy)
K – Kinetic energy [J]
W – Mechanical Work [J] $P = dW/dt$ = Power [W]
H – **Enthalpy** [J] (usually stored chemical energy)
Q – Heat energy [J]
η – Mechanical efficiency of a heat engine
Γ – Metabolic rate [W]

Physics Model of an Animal



Mass is Conserved!
Energy is Conserved!

In steady-state,

$\Delta M_{stored} = 0$ and $\Delta H_{stored} = 0$

- Mechanical efficiency
  $\eta = \Delta W_{out} / \Delta H_n$
- Resting Metabolic Rate Γ_0
  = *minimum* dH_n/dt *to stay alive.*

The Emergence of Mechanics

(a mathematical fantasy)

- Newton's Second Law: $F = m\,a = dp/dt \equiv \dot{p}$

  [Dot Notation for Time Derivatives]

- Time Integral: $\int F(t)\,dt = \Delta p$

  [Impulse changes Momentum]

- Dot Product with r & Path Integral: $\int F(r) \cdot dr = \Delta(\frac{1}{2} m v^2)$

  [Work changes Kinetic Energy]

- Cross Product with r: $r \times F \equiv \Gamma = r \times \dot{p} = \dot{L}$

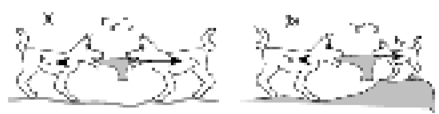
  [Torque changes Angular Momentum]

Poll: Within the context of *Classical Newtonian Mechanics*, assuming your weight is 600 N, approximately what ***net*** force do ***you*** exert *on the* ***Earth*** ?

a) 600 N upward
b) 0 N
c) 600 N downward
d) Other

Newton and the Free Body Diagram

Newton's Second Law: $\sum F = m\,a$

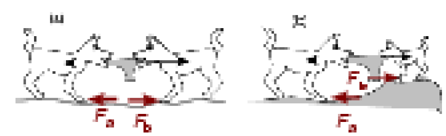


Doh! du jour

Not as simple as it sounds! **What** forces? *Mass* and acceleration of **what**? In the above picture, the "Free Body" is the (nearly massless) **sock** that the dogs are pulling on. Ergo $F_a = F_b$ almost exactly, or else the sock would have a huge acceleration!

Newton and the Free Body Diagram

Newton's Second Law: $\sum F = m\,a$



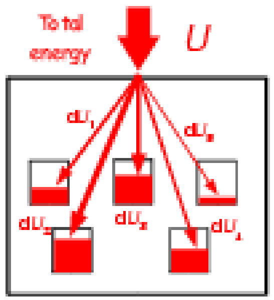
A **correct FBD** from which we can calculate the common acceleration of the entire system (both dogs plus the sock) involves the forces F_a and F_b exerted on the dogs' feet by **the ground**, in **reaction** to the forces the dogs exert with their feet.

(Newton's Third Law)

16/01/2007

Statistical Mechanics

An Abstract Introduction



The "System" is composed of many irreducible components, each of which can contain a share dU_i of the total energy U. There are many ways U can be distributed among all the components of the system. How many? Let's call the number $\Omega(U)$, since it will be a function of U. For any macroscopic system, Ω will be a big number, so let's take its natural logarithm. . . .

Statistical Mechanics

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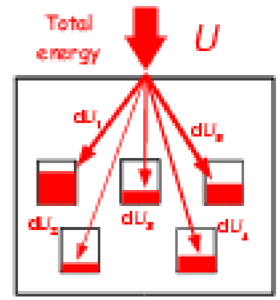

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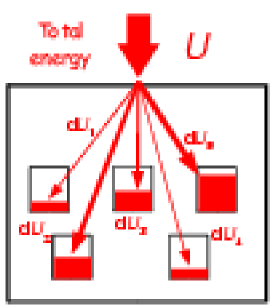
Entropy: $\sigma = \ln \Omega$



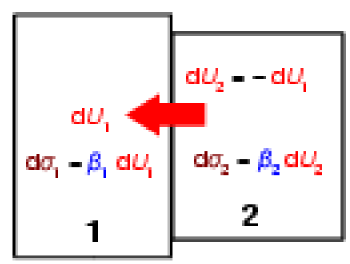
Remember, $\Omega(U)$ is the **number of ways a given total energy** U can be **distributed** among all the components of the system. Usually this goes up as U increases. So does the entropy σ. **How fast?**

Define $\beta = d\sigma/dU$.

Hold that thought.

Thermal Contact

The number Ω_1 of ways U_1 can be distributed within system 1 is independent of the number Ω_2 of ways U_2 can be distributed within system 2, so there are $\Omega = \Omega_1 \cdot \Omega_2$ ways that the total energy



Two Systems can exchange U.

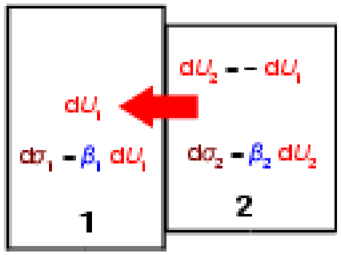
$U = U_1 + U_2$ can be distributed within the combined system. Thus the total entropy is $\sigma = \sigma_1 + \sigma_2$.

Since we assume these redistributions occur at random, the **most probable configuration** is one in which there are the most possibilities — the one with the **highest total entropy**.

Recall $\beta = d\sigma/dU$.

Thermal Equilibrium

Any exchange of energy (heat) that **increases the net entropy** produces a "macrostate" that is **more probable** than before, and so will **tend** to occur spontaneously through utterly random processes. How can we predict **whether heat will flow?** $d\sigma = d\sigma_1 + d\sigma_2 = \beta_1\, dU_1 + \beta_2\, dU_2$.



Recall $\beta = d\sigma/dU$.

but $dU_2 = -\, dU_1$, so

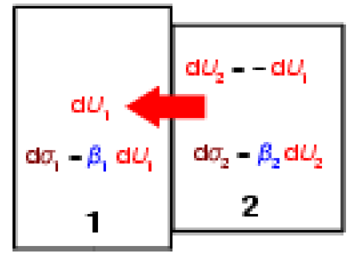
$$d\sigma = (\beta_1 - \beta_2)\, dU_1 .$$

When $\beta_1 = \beta_2$ a transfer of energy will have no effect on the total energy. This is called **thermal equilibrium**. It nicely corresponds to our notion of two systems having the **same temperature**.

Is β the temperature, then?

Cold & Hot

Any exchange of energy (heat) that **increases the net entropy** produces a "macrostate" that is **more probable** than before, and so will **tend** to occur spontaneously through utterly random processes. How can we predict **which way the heat will flow?** $d\sigma = d\sigma_1 + d\sigma_2 = \beta_1\, dU_1 + \beta_2\, dU_2$.



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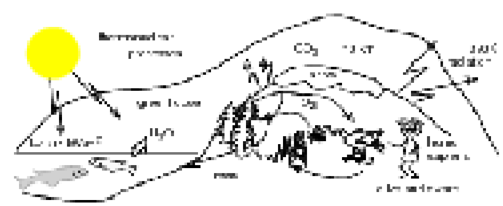
If $\beta_1 > \beta_2$ then transferring energy from 2 to 1 increases σ and will therefore happen spontaneously. This is what we expect to happen when 2 is **hotter** than 1 — implying that a **cold** system has a **larger** β than a **hot** system, opposite to our idea of "temperature". The solution is trivial. . . .

Temperature $\tau = k_B T$

The definition $\beta = d\sigma/dU = 1/\tau$ restores our "common sense" notion of **temperature**: a system with high τ is hot and will spontaneously give up heat to a cold (low τ) system. However, we must still deal with **units**. Since σ is a **pure number**, τ has units of energy (J). What happened to "degrees"? The answer is, "**Degrees are bogus!**" but we must live with bogosity, so Boltzmann invented a conversion constant: $k_B = 1.38066 \times 10^{-23}$ J/K (where K means "degrees Kelvin" which are the same **size** as °C but start 273.15° lower).

Likewise the conventional form of **entropy**: $S = k_B \sigma$

"Food" Chain



Energy **unit** conversions:

1 cal = 4.18 J 1 Cal = 1 kcal = 4.18 kJ

Thermal Radiation

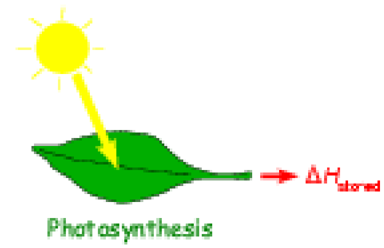
Look up "Insolation" on http://Wikipedia.org (great resource, but you can't use it as a formal reference, because it changes). The solar constant $S \approx 1370\ \mathrm{W/m^2}$ is "out in space" near Earth; we get hereabouts a bit less than $1\ \mathrm{kW/m^2}$ on a nice day.

At night, perfectly black surfaces at 0°C radiate about $0.3\ \mathrm{kW/m^2}$, of which a large fraction escapes into outer space on a clear night. Ask any farmer!

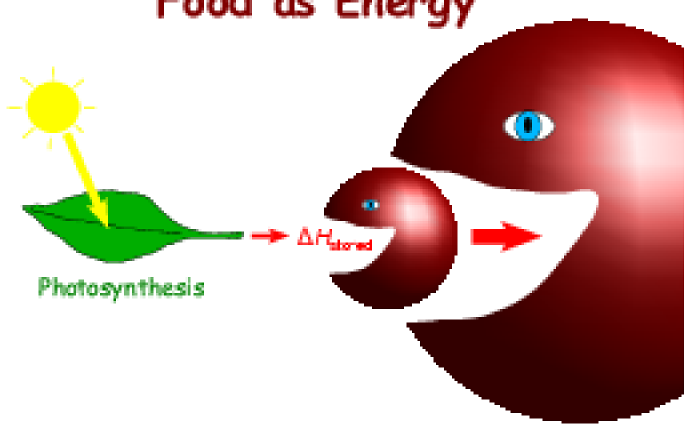
Stefan-Boltzmann Law: $P = \sigma_{SB} A T^4$

where $\sigma_{SB} = 5.67\times10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$. (Not an entropy!)

Energy Storage



Food as Energy

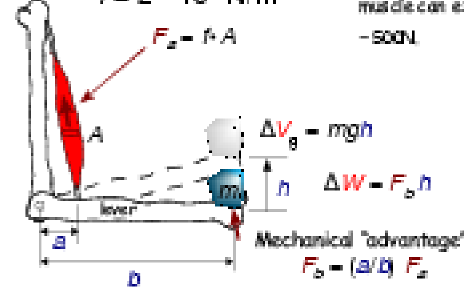


Muscle Work

Specific muscle stress

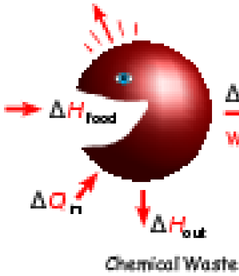
$f = 2 \cdot 10^5\ \mathrm{N/m^2}$

A typical biceps muscle can exert ~500N.



Thermal Regulation

$\Delta Q_{rad} = \Delta t \cdot A \cdot (\sigma_{SB} T^4 \approx 0.3$ kW per m^2 area)



$\Delta W = F \cdot \Delta r$

Work = force through a ‖ distance

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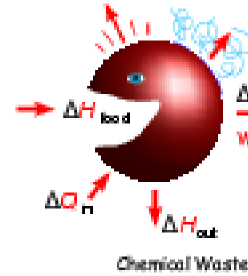

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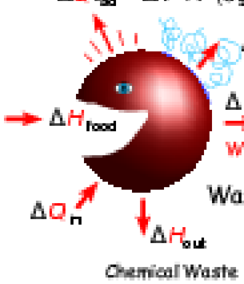

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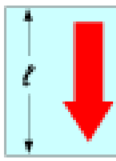
Work = force through a ‖ distance

Wait! You also get . . .

. . . (or **lose** heat through) **conduction** and **convection**!

Conduction of Heat

HOT side (T_H)



$\dot{Q}_{cond} = \kappa \cdot A \cdot (T_H - T_C) / \ell$

Thermal conductivity [W · m⁻¹ · K⁻¹]

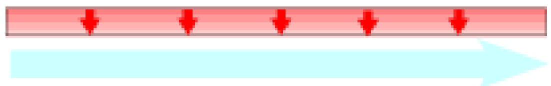
COLD side (T_C)

For an infinitesimal region in a thermal gradient,

$$J_U = \kappa \cdot \vec{\nabla} T$$

Convection: Heat Transport

Conduction across a ***thin*** *layer can be very efficient, but the heat must be* ***taken away*** *on the other side!*



This requires cool mass flow past a warm surface.

$\dot{Q}$ will be proportional to the area of the contact surface, the rate of flow and the specific heat of the flowing mass, and inversely proportional to the thickness of the fixed layer between hot and cold surfaces (since conduction is still necessary).