THE UNIVERSITY OF BRITISH COLUMBIA

Physics 438 Assignment # 3:
FLUID MECHANICS & LOCOMOTION
SOLUTIONS:

Tue. 06 Feb. 2007 — finish by **Thu. 15 Feb.**

Please hand in one assignment per group and list the names & Email addresses of all group members at the top of each sheet. In general, *if you think some necessary information is missing, make a reasonable assumption.* But always *write down* what that assumption is. Always estimate your *uncertainty* in any measured quantity, and don't forget to specify all *units*. If possible, justify your input. For original comments you may score bonus points!

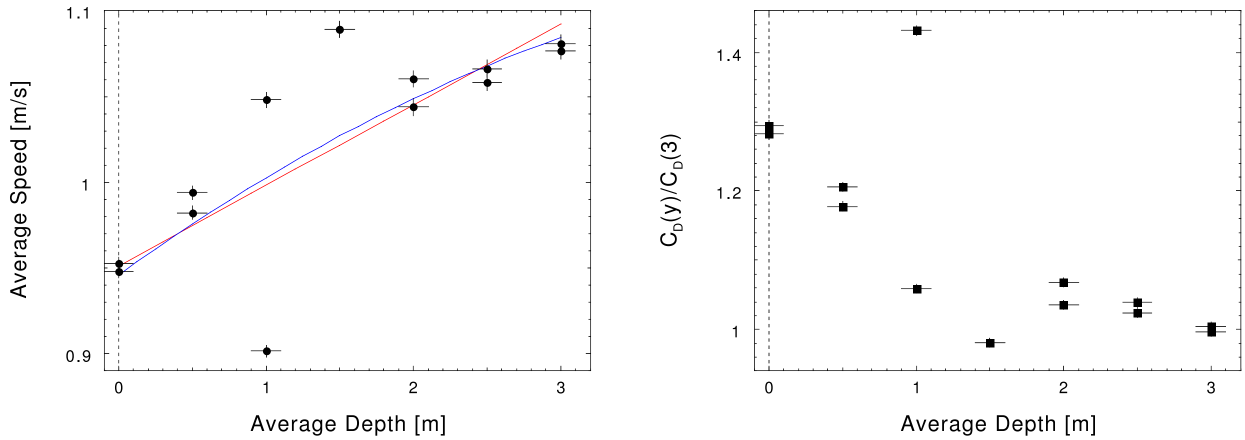
1. NEAR SURFACE DRAG: (project by Natasha Szucs, April 2002) Natasha, a good swimmer, wants to quantify the effect of near surface drag. For that purpose she swims with the dolphin kick at various depths y under the water surface. She maintains a constant depth by watching a horizontal line on the pool wall. For every length she swims she measures her travel time with a stopwatch on her wrist.[1] The UBC pool is slightly shorter below a depth of 1.50 m (see Δx in table below). She also measures her pulse rate F_h, and only uses runs where it stays close to the same value, 140 beats per minute.

Table 1 : Natasha's distance Δx, elapsed time Δt and pulse rate F_h while swimming at depth y under water.

y [m]	Δx [m]	Δt [s]	F_h [beats/min]	avg. speed [m/s]	ratio C_{D_y}/C_{D_3}
0	25	26.24	140	0.953	1.282
0	25	26.37	136	0.948	1.295
0.5	25	25.45	140	0.982	1.206
0.5	25	25.15	140	0.994	1.178
1.0	25	23.85	140	1.048	1.059
1.0	25	27.73	140	0.902	1.432
1.5	25	22.95	144	1.089	0.981
1.5	25	22.95	140	1.089	0.981
2.0	22.86	21.90	140	1.044	1.068
2.0	22.86	21.56	144	1.060	1.035
2.5	22.86	21.60	140	1.058	1.039
2.5	22.86	21.44	140	1.066	1.024
3.0	22.86	21.15	136	1.081	0.996
3.0	22.86	21.23	140	1.077	1.004

(a) Calculate the average speed for each lap. **ANSWER**: $u = \Delta x/\Delta t$. See the table above and the graph below.

[1] See Table 3.12, p. 125.



LEFT: Average speed as a function of depth. Plausible uncertainties ("error bars") have been included and χ^2-minimization fits shown as lines: the straight line represents a first order polynomial fit ($u = 0.951 + 0.047y$) and the curved line represents a second-order polynomial fit ($u = 0.946 + 0.062y - 0.0053y^2$). RIGHT: Depth dependence of the drag coefficient relative to that at a depth of 3 m.

(*b*) Assume a body shape like a flattened torpedo of cross section area $A = 0.07$ m^2, and a drag coefficient $C_{D_3} = 0.05$ to calculate the average drag force F_3 at a depth of $y = 3.0$ m. This is also the average propulsion force generated by the swimmer. **ANSWER**: The Reynolds number for such a shape is given by $Re = D\,u/\nu$, where $D \approx 0.3$ m is the diameter of the torpedo, $u \approx 1$ m/s is a typical velocity and $\nu \approx 10^{-6}$ m^2/s is the kinematic viscosity of water (from Table **3.4** on p. 86 of the textbook). Thus $Re \approx 3 \times 10^5$ and the flow is *very* turbulent, so that most of the drag is *hydrodynamic drag* and we can ignore laminar drag (skin friction). According to Fig. **3.18** on p. 92 of the textbook, this is just the range of Re in which the drag coefficient C_D suddenly begins to drop with increasing Re. Therefore we should expect dramatic differences between swimming speeds of athletes with only slightly different thrust. However, the depth dependence is primarily due to *ventilation drag* and/or *wave drag*, as shown in Fig. **3.22** on p. 95. The swimmer (even underwater) creates a wave that travels with her, and must push along the "hump of water" that results; the size of this "hump" decreases as she gets further from the surface. Once she is far enough below the surface that we can ignore this effect, we should be able to use Boye's formula on p. 93 for the hydrodynamic drag: $F_h = (1/8)C_D \times \pi D^2 \rho u^2$ with $C_D \approx 0.03$. This gives $\boxed{F_h \approx 1.2 \text{ N}}$ at $u_3 = 1.066$ m/s.

(*c*) Since the heart rate is about the same at all depths one can assume that the propulsion force too is the same at all depths. Calculate the drag coefficient ratio C_{D_y}/C_{D_3} as function of depth y. **ANSWER**: In unaccelerated motion at constant velocity, the propulsion force is exactly balanced by the equal and opposite drag force, so if one is constant, so is the other. Since $F_h \propto C_D \times u^2$ (everything else is constant), a constant F_h implies $C_D \propto u^{-2}$ or $\boxed{C_{D_y}/C_{D_3} = (u_3/u_y)^2}$. This can be used to calculate the ratio as a function of y. See the table and graph above. Note however that the data "jump around" a lot, indicating large systematic uncertainties (hardly surprising) so we would be foolish to take the simple linear or second order polynomial fits too seriously. All we can really conclude is that "deeper tends to be better" from the standpoint of the drag coefficient, and possibly that the improvement levels off after the first meter or two.

(*d*) What advice should the UBC coach give to the swimmers? **ANSWER**: (*i*) Be grateful you're in a sport for which UBC provides a facility — as opposed to (for instance) Track & Field, for which UBC alone, *among all the major Universities in the civilised world*, has **no** facility (track) whatsoever! (*ii*) On the start, always swim underwater for as far and as deep as rules allow. (There are strict limits on this, for obvious reasons.) (*iii*) Natasha may decrease her lap time by swimming at a greater depth, but of course she must return to the surface to breathe. There is also another reason to stay at the surface: air is thinner than water! A swimmer may try to propel herself not only forward, but also upward, so that her shoulders are only partly in the water, thereby reducing the frontal area pushing against the water and therewith the drag force.[2] In addition, the wave itself may be utilized to a certain extent, especially with butterfly and breast strokes. If you observe competitive swimmers

[2] In effect, the swimmer is trying to emulate the water-walking basilisk lizard!

you will always see an upward and downward oscillation, which is partly used to overcome the wave, especialy in the butterfly.

Surface waves aren't bad for everyone. The largest whales may be able to harness the energy from waves for their own propulsion[3] and dolphins are able to save energy by riding waves.[4]

Humans are not as well-streamlined as marine mammals: the drag coefficient for a towed human is nearly 3.5 times greater than a towed harbor seal.[5]

2. RED FINGERS: Swirl one arm around as fast as you safely can. Measure the length L of your arm and the time Δt_{10} it takes for 10 revolutions, so that you can determine the period τ.

(*a*) Calculate the average speed of your fingers. **ANSWER**: Assuming that you hold your shoulder fixed and are flexible enough to swing your arm in a circle, you can use $v = r\omega$ with $r = L$ and $\omega = 2\pi/\tau$ to calculate v. For a typical $L = 0.8$ m and $\tau = 0.6$ s this would give $v = 8.4$ m/s.

(*b*) Determine the radial centrifugal acceleration and the additional pressure in the blood vessels in your fingers due to the motion. Compare this pressure to the systolic pressure generated by your heart ($\Delta p_h \approx 120$ mm Hg), and comment why your fingers are red. **ANSWER**: The radial acceleration of your fingers is given by $a = r\omega^2$ or $a = v^2/r$. For the typical values above, this would give $a = 88$ m/s^2. Compare the acceleration of gravity, $g = 9.81$ m/s^2; it is as if your arm were hanging down in a gravitational gradient that starts at zero (at your shoulder) and increases linearly to $8.9g$ at your fingers. Because the "artificial gravity" is not constant, you can't just multiply $\rho g L$ by 8.9 to get the addidtional hydrostatic pressure at your fingers; you must use the *average* a (which, because a changes linearly with r, is just half the value at your fingers) or take the integral of $a(r)\rho dr$, which amounts to the same thing. The result (for the above typical values) is $p = 35,000$ N/m^2, or about 1/3 of an atmosphere, or over $2\frac{1}{2}$ times the typical systolic blood pressure of 13,300 Pa. No wonder your fingers are red! If the same blood pressure were produced in your *head* for any length of time, you would be in grave danger of an aneurysm or stroke. Fortunately the blood vessels in your extremities were designed to handle such extremes.

3. HOW DOES THE FLEA GET YOU? A flea can be modeled as a sphere of 1 mm diameter with a density close to that of water. The flea accelerates at an average rate of 200 g (≈ 2000 m/s^2), achieving a takeoff velocity that allows it to reach a potential host at $h = 0.35$ m above ground.

(*a*) What is the takeoff velocity? **ANSWER**: First let's assume that air drag is negligible, in which case the initial kinetic energy $K_0 = \frac{1}{2}mv_0^2$ must equal the gravitational potential energy mgh at the top of the (presumed vertical) trajectory, *i.e.* $v_0 = \sqrt{2gh}$ or $\boxed{v_0 = 2.62 \text{ m/s}}$. Now let's check to see if it is reasonable to neglect drag. The Reynolds number of a sphere of $D = 10^{-3}$ m moving at $u = 2.62$ m/s through a medium with kinematic viscosity $\nu = 16 \times 10^{-6}$ m^2/s is $Re = Du/\nu = 164$ so the flow of air past the flea is laminar throughout and the Stokes friction force is $F_D = 6\pi r\eta u$, where $\eta = 18 \times 10^{-6}$ kg·s^{-1}·m^{-1} is the viscosity of air and $r = 0.5 \times 10^{-3}$ m, giving $F_D = 0.44 \times 10^{-6}$ N initially. Thus the *de*celeration due to air friction is $a_f = F_D/m$ where $m = \frac{4}{3}\pi r^3 \rho = 0.52 \times 10^{-6}$ kg, or $a_f = 0.85$ m/s^2 — about $0.087g$. This additional deceleration is a small fraction of the deceleration due to gravity, and only gets smaller as u decreases, so it is not a bad approximation to neglect air friction – certainly our calculated v_0 is off by less than 10%.

(*b*) How long is the acceleration phase? **ANSWER**: Assuming a constant acceleration of $a_0 = 2000$ m/s^2, $v_0 = a_0 t_0$ gives $\boxed{t_0 = 0.00131 \text{ s} = 1.31 \text{ ms}}$.

(*c*) What force is required to obtain this acceleration? **ANSWER**: We calculated $m = 0.52 \times 10^{-6}$ kg earlier; thus $F_0 = ma_0$ gives $\boxed{F_0 = 1.047 \times 10^{-3} \text{ N}}$.

(*d*) What is the power (force × velocity) at takeoff? **ANSWER**: Since (assuming constant acceleration) the velocity is initially zero and grows linearly with time to a final value of v_0, the power is initially zero and grows linearly with time to a final value of $F_0 v_0$. The *average* power is thus half the final power, or $\boxed{\langle P_0 \rangle = 1.37 \times 10^{-3} \text{ W}}$.

[3] Bose and Lien, "Energy Absorption from ocean waves: A free ride for cetaceans", Proc. Roy. Soc. Lond. B. **240**, 591-605 (1990).

[4] Williams *et al.*, Nature (1992).

[5] T.M. Williams and G.L. Kooyman, "Swimming performance and hydrodynamic characteristics of harbor seals *Phoca vitulina*", Physiological Zoology **58**, 576-589 (1985).

(*e*) Make a reasonable assumption about the muscle mass involved in a jump and calculate the power requirement per kg of muscle. Given that a typical muscle output is ≈ 100 W/kg, comment on your answer. **ANSWER**: Let's be generous and assume 1/5 of the entire mass of the flea is muscle used in the jump. That would be about 10^{-7} kg. The power-to-mass ratio would then be $\boxed{\text{14,000 W/kg}}$. Even if the flea were *all* muscle, we would get 2620 W/kg, which is a little implausible! There must be something else going on. Oh yes...

(*f*) Before a jump, the flea stores energy in a pad of resilin, a rubber-like protein built into each hind leg, about 30 μm thick and 80 μm in diameter, with an energy storage capacity of about 1.5×10^6 J/m^3. How much potential energy could the flea store in each pad? How long would it take the flea to put that much energy into each pad? **ANSWER**: The volume of a disc 30 μm thick and 80 μm in diameter is 0.151×10^{-12} m^3, so it could store $\boxed{0.226 \times 10^{-6}\ \text{J}}$. Assuming that the "mouse to elephant" allometric relation holds, the flea's resting metabolism is about $\Gamma_0 = 4m^{3/4} \approx 0.78 \times 10^{-4}$ W. If the flea stores energy in its resilin pads at roughly this rate, it will take about $\boxed{\text{0.0029 s or 2.9 ms}}$ for the flea to store up enough energy for another jump. (Presumably it will eventually get tired if it keeps this up, but it can obviously jump pretty often!)