# The Exponential Function — Reference Sheet

- **Essential Property:** The rate of change [slope] of the function is proportional to the function itself:

$$\frac{dy}{dx} = k\, y$$

- **Solution:** The function satisfying this property is

$$y(x) = y_0\, e^{kx} \qquad [y_0 \text{ is the value of } y \text{ at } x = 0]$$

$$\text{Thus} \qquad \frac{d(e^{kx})}{dx} = k\, e^{kx}$$

Note: the derivation on the handout defines the function $\exp(x)$ in terms of a power series, but it does not justify the claim that this function is equivalent to the number $e \equiv 2.718\cdots$ raised the the $x^{\text{th}}$ power, of which we make much use. Consult your Math text for a proof of this equivalence.

- **Power of a Product:** $e^{kx} = [e^x]^k$ so it suffices to consider just the simplest case, $e^x$

- **Negative Powers:** $e^{-x} = \dfrac{1}{e^x}$

- **Limits:**

$$e^0 = 1$$

$$e^{x\to\infty} \longrightarrow \infty$$

$$e^{x\to-\infty} \longrightarrow 0$$

- **Natural Logarithms:**

$$e^{\ln(x)} = x \qquad \ln(e^x) = x \qquad \frac{d[\ln(x)]}{dx} = \frac{1}{x}$$

- **Ratios:**

$$e^{-x} = \frac{1}{2} \qquad \text{when} \qquad x = \ln(2) = 0.693$$

$$e^{-x} = \frac{1}{4} \qquad \text{when} \qquad x = \ln(4) = 2\ln(2) = 2 \times 0.693 = 1.386$$

  - $e^{-x}$ decreases by a (multiplicative) factor of $\frac{1}{2}$ each time the "argument" $x$ increases by an (additive) amount 0.693
  - $e^{-x}$ decreases by a multiplicative factor of $\frac{1}{e} = 0.368$ each time the "argument" $x$ increases by 1.